Electromagnetic Field with Constraints and Papapetrou Equation

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It is shown that a geometric optical description of the electromagnetic wave with respect to its polarization in a curved space-time can be obtained straightforwardly from the classical variational principle for the electromagnetic field. For this purpose the entire functional space of electromagnetic fields must be reduced to its subspace of locally plane monochromatic waves. We have formulated the constraints under which this can be achieved. These constraints introduce variables of another kind which specify a field of local frames associated with the wave. They contain some congruence with null-curves. The Lagrangian for constrained electromagnetic fields contains variables of two kinds, namely a congruence of null-curves and the field itself. This in turn yields two kinds of Euler-Lagrange equations. The equations of the first kind are trivial due to the constraints imposed. The variation of the curves yields the Papapetrou equations for a classical massless particle with helicity 1.